Logs Basics – Notes

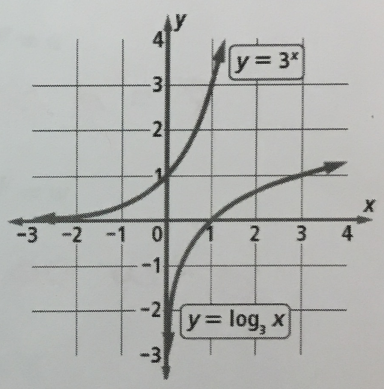
Name: KEY Date:

Common Logs

Definition of Logarithm of x to the Base 10

y IS THE LOGARITHM OF x TO THE BASE 10, WRITTEN $y = \log_{10} x$, IF AND ONLY IF $10^y = 10$.

Exponential Form	Logarithmic Form
$10^7 = 10{,}000{,}000$	$\log_{10} 10{,}000{,}000 = 7$
$10^{-3} = 0.001$	$\log_{10} 0.001 = -3$
$10^{\frac{1}{2}} = \sqrt{10}$	$\log_{10} \sqrt{10} = \frac{1}{2}$
$10^{-\frac{1}{4}} = \frac{1}{\sqrt[4]{10}}$	$\log_{10} \frac{1}{\sqrt[4]{10}} = 10^{-\frac{1}{4}}$
$10^a = b$	$\log_{10} b = a$



* The log of x is the exponent to which THE BASE IS RAISED TO GET x

* Logs to the base 10 are called COMMON LOGS

Evaluating Logs With a Calculator

a) $\log \sqrt{2} \approx$ 0.1505

b) $\log 5 \approx$ 0.699

Examples: Rewrite each equation in exponential form.

1) $\log_{10} y = -16$

$10^{-16} = y$

2) $\log_{10} w = 5$

$10^{5} = w$

3) $\log_{10} 8 = a$

$10^{a} = 8$

4) $\log_{10} -3 = x$

$10^{x} = -3$

Examples: Rewrite each equation in logarithmic form.

1) $10^{-5} = d$

$\log d = -5$

2) $10^{9} = a$

$\log a = 9$

3) $10^{x} = 5$

$\log 5 = x$

4) $10^{b} = w$

$\log w = b$

Logs to bases other than 10

Definition of Logarithm of a to the Base *b*

LET $b > 0$ AND $b \neq 1$. THEN x IS THE LOGARITHM OF a TO THE BASE b, WRITTEN $x = \log_b a$, IF AND ONLY IF $b^{x} = a$.

Examples: Rewrite each equation in exponential form:

1) $\log_7 49 = y$

$7^y = 49$

2) $\log_8 2 = a$

$8^a = 2$

3) $\log_b\left(\frac{1}{64}\right) = 4$

$b^4 = \frac{1}{64}$

4) $\log_4 h = \frac{3}{2}$

$4^{3/2} = h$

Examples: Rewrite each equation in logarithmic form:

1) $2^f = 8$

$\log_2 8 = f$

2) $g^5 = 3$

$\log_g 3 = 5$

3) $-8^a = b$

$\log_{-8} b = a$

4) $x^y = z$

$\log_x z = y$

Change of Base Theorem:

$$\log_a b = \frac{\log b}{\log a}$$

Examples: Evaluate & round to the nearest hundredth.

1) $\log_3 4.2 =$

$\frac{\log 4.2}{\log 3} = \frac{.623}{.477}$

$= 1.306$

2) $\log_2 5$

$\frac{\log 5}{\log 2} = \frac{.69897}{.30103}$

$= 2.322$

Unit 4

Natural Logs

$\star e \approx 2.71828$

Definition of Natural Logarithm of m

n IS THE NATURAL LOGARITHM OF m, WRITTEN $n = \ln m$, IF AND ONLY IF $m = e^n$.

Examples: Rewrite each expression with natural logs.

1) $\log_e 7$ — $\ln 7$

2) $\log_e -5$ — $\ln -5$

3) $\log_e 0.62$ — $\ln 0.62$

Examples: Rewrite each expression in exponential form.

1) $\ln 9 = 2.2$ — $e^{2.2} = 9$

2) $\ln 4.7 = 1.55$ — $e^{1.55} = 4.7$

3) $\ln 2.68 = 0.986$ — $e^{0.986} = 2.68$

Examples: Rewrite each expression in logarithmic form.

1) $e^2 = 7.389$ — $\ln 7.389 = 2$

2) $e^{-8} = 0.0003$ — $\ln 0.0003 = -8$

3) $e^{1.05} = 2.858$ — $\ln 2.858 = 1.05$

Examples: Use your calculator to estimate each to the nearest thousandth:

1) $\ln 100$ — 4.605

2) $\ln 5$ — 1.609